

STORM SURGE PREDICTION MODEL
AND ITS APPLICATION TO A
COMBINED TRUNK SEWER SYSTEM

BY

A.K. BHATTACHARYYA¹, D.L. FREAD²

¹Senior Hydrologist, Associated Engineering Services Ltd. (AESL),
Edmonton, Alberta.

²Senior Research Hydrologist, Hydrologic Research Laboratory, U.S.
National Weather Service, Silver Spring, Maryland.

ABSTRACT

Hydraulics of unsteady flow in storm sewers are investigated by using a rapidly varying dynamic wave model. The Storm Surge Prediction model (acronym SSP), being developed at AESL, solves the complete dynamic equations using the "weighted four-point" implicit scheme for open channel as well as for surcharged conditions. One example simulation of a combined trunk in the City of Edmonton is presented. The pressure waves created by the introduction of inflows at surcharged nodes have been shown to have significant influence on the peak water level of the overloaded trunk system.

INTRODUCTION

Many large cities in Canada and United States are serviced by combined deep tunnels. These systems are overloaded due to continuous urbanization over the years. Municipalities have to annually spend millions of dollars to restore these systems to acceptable service levels. Prior to undertaking any intensive capital works, the existing performance level and the proposed remedial measures should be evaluated by computer based mathematical models capable of simulating the drainage system operation. The above problem consists of two major parts: the first is to generate representative urban storm runoff histograms, and the second is to route these flow histograms through the underground trunk networks. This paper concentrates on the dynamic routing of the flow histograms through underground trunk networks.

Flow in major trunk sewer systems is usually unsteady, non-uniform and rapidly varying in time. To date, the SWMM EXTRAN BLOCK has been extensively used to simulate the hydraulic response of overloaded trunk systems. The SWMM EXTRAN BLOCK is a node and link model, based on an explicit scheme (1). The scheme predicts conduit flows by the Saint-Venant equation of motion, according to the system properties, based on Modified Euler Solution Method. The continuity is preserved at nodes by changing head, i.e., accounting for the storage volume around the node and subtracting the projected nodal outflows over the integration period. Since in-system storage does not change at the surcharged nodes, the continuity equation is modified to account for the change of discharge with respect to head at the surcharged nodes. The inflows to the surcharged nodes are equated with the outflows by an iteration scheme with the equation of motion. The method requires very short computational

time intervals and at times is unstable (1).

The Storm Surge Prediction model (acronym SSP) is being developed at AESL to trace the path of storm surge propagation along the trunks for free discharge as well as for surcharged conditions. The basic structure of the model has been adopted from the U.S. National Weather Service's Dam-Break Model (acronym DAMBRK) with the appropriate modification for circular conduit elements in the solution scheme. The basis of this model is a four-point implicit scheme first used by Preissmann (2) and Amein (3) and later modified by Fread (4,5,6,7) to simultaneously solve the Saint-Venant equations of motion and continuity for applications to rapidly varying flow conditions. The purpose of this paper is to illustrate the four-point implicit scheme as modified for circular conduits in SSP. It is intended that this scheme will be applied later to the NWS FLDWAV model (8) which will allow treatment of Sewer Networks.

The theoretical basis for the unsteady flows in circular conduits are formulated for free flow, as well as for surcharged conditions. One trunk system example simulation by SSP is presented. The results have been compared with that of the SWMM EXTRAN model.

MATHEMATICAL BASIS

The basis for SSP is a finite difference solution of conservation form of the one-dimensional equations of unsteady flow consisting of conservation of mass and momentum equations, i.e.,

$$\partial A / \partial x + \partial A / \partial t - q = 0$$

$$\partial Q / \partial t + \partial (Q^2 / A) / \partial x + g A (\partial h / \partial x + S_f) - q V_x = 0 \quad (2)$$

in which Q is discharge, A is cross-sectional area, q is lateral inflow or outflow, x is the distance along channel, t is time, g is gravity acceleration constant, V_x is the velocity of lateral inflow in the x -direction and S_f is the frictional slope defined as:

$$S_f = n^2 |Q| / (2.2 A^2 R^{4/3}) \quad (3)$$

in which n is the Manning's roughness coefficient and R is the hydraulic radius. The solution of equations (1) and (2) in SSP is based on a "weighted four-point" implicit scheme investigated by Fread (4). The advantage of this scheme is that it can readily be used with unequal distance steps and its stability-convergence criteria can be controlled. Also, the inherent numerical stability properties of the implicit scheme allow larger time steps than explicit solution schemes. In the weighted four-point scheme, the continuous x - t region in which solutions of h and Q are sought is represented by rectangular net of discrete points, as shown in Figure 1, at equal or unequal intervals of Δx and Δt along the x and t axes, respectively. Each point is identified by a subscript (i) which designates the x position and a superscript (j) for the time position. The time derivatives are approximated by:

$$\frac{\partial K}{\partial t} \approx (K_i^{j+1} + K_{i+1}^{j+1} - K_i^j - K_{i+1}^j) / 2 \Delta t \quad (4)$$

in which K represents any variable. The spatial derivatives are approximated by a finite difference quotient positioned between two adjacent time lines according to weighting factors θ and $1 - \theta$, i.e.,

$$\frac{\partial K}{\partial x} \approx \theta / \Delta x (K_{i+1}^{j+1} - K_i^{j+1}) + (1 - \theta) / \Delta x (K_{i+1}^j - K_i^j) \quad (5)$$

and variables other than derivatives are approximated in a similar manner i.e.,

$$K \approx \theta / 2 (K_i^{j+1} + K_{i+1}^{j+1}) + (1 - \theta) / 2 (K_i^j + K_{i+1}^j) \quad (6)$$

When θ equals 1.0, a fully implicit scheme is formed. A box scheme results if θ is fixed at 0.5. The influence of the θ weighting coefficient factor on the stability and convergence properties for open channel flow was examined by Fread (4), who concluded that accuracy decreases as θ increases from 0.5 and approaches 1.0. A θ value of 0.6 has been assumed for the SSP testing.

Substitution of the finite difference quotient defined by equations (4), (5) and (6) into equations (1) and (2) for the derivative and nonderivative terms produces two algebraic equations with respect to the unknowns h and Q at the net points on the $j + 1$ time line. All terms associated with the j th time line are known from either the initial conditions or previous computations. Similar equations are formed for each of the $N-1$ Δx reaches between the upstream and downstream boundaries, a total of $2N-2$ equations with $2N$ unknowns results. The prescribed boundary conditions, one at the upstream extremity of the trunk and one at the downstream extremity, provide the necessary two additional equations required for the system to be determinate. The resulting system of $2N$ nonlinear equations with $2N$ unknowns is solved by a functional iterative procedure based on the Newton-Raphson Technique (5).

The adoption of the above noted solution technique involved the open channel approximation of a closed circular conduit, as shown in Figure 2. The virtual chimney (9) is conceived to contain the additional fluid volume, as expanded at the elastic circular section during upsurge and at the vertical manhole shafts during sewer surcharge. During downsurge, the chimney provides the necessary volume to fill in the void at the circular section.

The derivative and nonderivative variables of interest are:

$$\phi = \pi + 2 \sin^{-1}(\gamma - \tau/\tau) \quad \text{if } \gamma \leq 0 \quad (7)$$

$$\phi = 2\pi \quad \text{if } \gamma > 0 \quad (8)$$

$$A = D^2/8 (\phi - \sin \phi) \quad \text{if } \gamma \leq 0 \quad (9)$$

$$A = \pi D^2/4 + fD \quad \text{if } \gamma > 0 \quad (10)$$

$$B = D \sin(\phi/2) \quad \text{if } \gamma \leq 0 \quad (11)$$

$$B = fD \quad \text{if } \gamma > 0 \quad (12)$$

$$\bar{R} = \bar{A}/\bar{P} \quad (13)$$

$$d\phi_i/dy_i = 2/(\tau + \sqrt{1 - (\gamma - \tau/\tau)^2}) \quad \text{if } \gamma \leq 0 \quad (14)$$

$$d\phi_i/dy_i = 2/\tau \quad \text{if } \gamma > 0 \quad (15)$$

$$dB_i/dy_i = D/2 * d\phi_i/dy_i * \cos(\phi/2) \quad \text{if } \gamma \leq 0 \quad (16)$$

$$dB_i/dy_i = 0 \quad \text{if } \gamma > 0 \quad (17)$$

$$dP_i/dy_i = \tau * d\phi_i/dy_i \quad (18)$$

$$dA_i/dy_i = D^2/8 (d\phi_i/dy_i - d\phi_i/dy_i * (\cos \phi)) \quad \text{if } \gamma \leq 0 \quad (19)$$

$$dA_i/dy_i = fD \quad \text{if } \gamma > 0 \quad (20)$$

$$d\bar{R}/dy_i = \bar{R}/3 \left(\frac{dA_i/dy_i}{\bar{A}} - \frac{dP_i/dy_i}{\bar{P}} \right) \quad (21)$$

$$d\bar{S}_b/dy_i = 2 \bar{S}_b \left[\frac{d\bar{n}/dy_i}{2\bar{n}} - \frac{dA_i/dy_i}{2\bar{A}} - \frac{d\bar{R}/dy_i}{\bar{R}} \right] \quad \text{if } \gamma \leq 0 \quad (22)$$

$$d\bar{S}_b/dy_i = 0 \quad \text{if } \gamma > 0 \quad (23)$$

where f has been set to 0.002 in this investigation.

Equations (7-23) are derived from the properties of circular conduit with a virtual chimney as identified in Figure 2. The sectional variables are also identified in this figure. Whereas nonderivative terms, defined by equations (7-13) are determined by y at a known time level, the derivative terms, defined by equations (14-23) are used in the Newton-Raphson functional iterative procedure.

EXAMPLE COMBINED TRUNK SYSTEM

The example of a combined trunk system, as shown in Figure 3, is based on the City of Edmonton 105 Street deep tunnel from 51 Avenue and along 80 Avenue to Mill Creek outfall. The storm inflows into the system, as shown in Figure 3(b), have been idealized on the basis of 5-year flows as simulated by SWMM to relieve the local combined drains up to the deep tunnel (10). The inflows linearly increases from a total base flow of 155 cfs at time $t = 0.2$ hour to a peak flow of 437 cfs at time $t = 0.5$ hour. The peak flow duration is for 0.3 hour for the system to reach steady state at time $t = 0.8$ hour. The peak flow of 437 cfs at $t = 0.8$ hour recesses to the base flow of 155 cfs at time $t = 1.1$ hour. The system, therefore, is subjected to a transient condition from time $t = 0.2$ hour to 0.5 hour. The subsequent period of 0.3 hour is allowed for the system to reach steady state. In the following period of 0.3 hours, the transient conditions bring the system back to the initial state. The assumed Mill Creek outfall rating is shown on Figure 3(c).

SSP TESTING

The trunk system has been schematized for the SSP application by 44 cross-sections, i.e. circular pipe with an imaginary chimney of width 0.02 times diameter rising to the atmosphere. The minimum cross-section interval for interpolation was set at 50 feet in the vicinity of junctions. At other locations the interval varies according to the location of lateral inflows and variation of the bottom profile. The total number of computational points is 71. Manning's 'n' was assumed to be 0.015. The weighting factor θ was

assumed to be 0.60. The computational time interval was set at 0.005 hour and 0.01 hour for testing the numerical scheme.

The response of the trunk system as simulated by SSP for a computational time step of 0.01 hour is presented on Figure 4. The trunk system is at capacity at time $t = 0.62$ hour based on available depth of flow. The corresponding outflow at Mill Creek is 267 cfs which compares well with the conduit capacity of 264 cfs at the outlet pipe. The cumulative inflows at time $t = 0.61$ hour is 437 cfs. The difference of 273 cfs between inflow and outflow is stored in the pipe system. The hydraulic grade line at time $t = 0.68$ hour indicates a pressure rise at the junction of the 5.5-foot and 6.5-foot pipes. The downstream pipe segment outflows at a surcharged capacity of 368 cfs. The difference of 69 cfs between inflow and outflow causes the maximum upsurge pressure at the junction of 5.5 foot and 6.5 foot pipes. The height of this pressure head is about 9 feet above the head at upstream. At time $t = 0.69$ hours, the heavily surcharged response is explained by the propagation of the pressure head upstream increasing the outflow rate to 387 cfs. The peak water level of 217.3 feet at the upstream boundary occurs at this time. At time $t = 0.71$ hour, the response is very similar with an outflow rate of 451 cfs as the pressure head propagates downstream. At time $t = 0.80$ hour, the system reaches steady state.

The nature of the pressure wave propagation, as well as the accuracy of computation by SSP, has been examined by simulating the system response for a computational time interval of 0.005 hour. The propagation of the pressure head is very similar in nature. However, at time $t = 0.6750$ hour, the maximum height of the pressure head above the upstream end is 14 feet. Outflow rate

at this time is 377 cfs into Mill Creek. The peak water level of 212.45 feet occurs at time $t = 0.715$ hour outflowing at a rate of 451 cfs into Mill Creek. Therefore, the surcharged water level is somewhat sensitive to the computational time interval as well as the assumed width of chimney.

COMPARISON OF PEAK PROFILE BY SSP AND EXTRAN

The response of the trunk system has been simulated by the EXTRAN model for comparing SSP results. Peak water levels by SSP and EXTRAN have been compared in Figure 5. The maximum difference in peak water level along the system is 17.5 feet. The difference is explained by the fact that EXTRAN relies only on the nodal water level along the links to compute outflows by Manning's equation. Therefore, the simulated response, by EXTRAN, for this example, is much slower with a peak outflow of 346 cfs into Mill Creek occurring at time $t = 0.902$ hours.

In Figure 5, the peak water level by SSP is also compared with the steady-state profile computed by Manning's equation. The maximum difference of 7 feet between SSP peak and steady state profile should be expected due to the propagation of pressure waves through the system.

CONCLUSION

The example presented in this paper demonstrates that unsteady open-channel, as well as surcharged flow in trunk sewers, can be routed by using the "weighted four-point" implicit scheme as modified for the SSP model. The flow simulation results indicate that the storm surge can be severe in overloaded

trunk sewer systems. Additional study is needed to determine appropriate values for the time step and chimney width, a priori in a given application.

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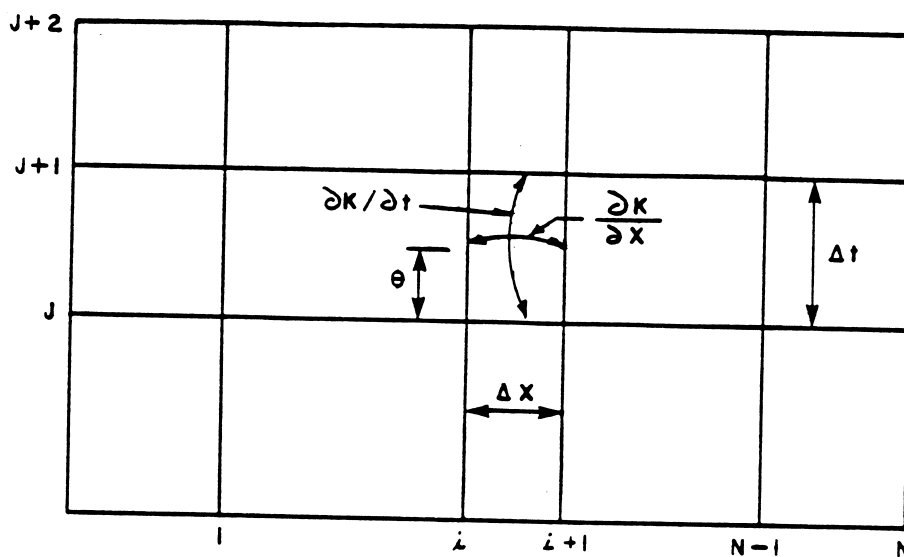
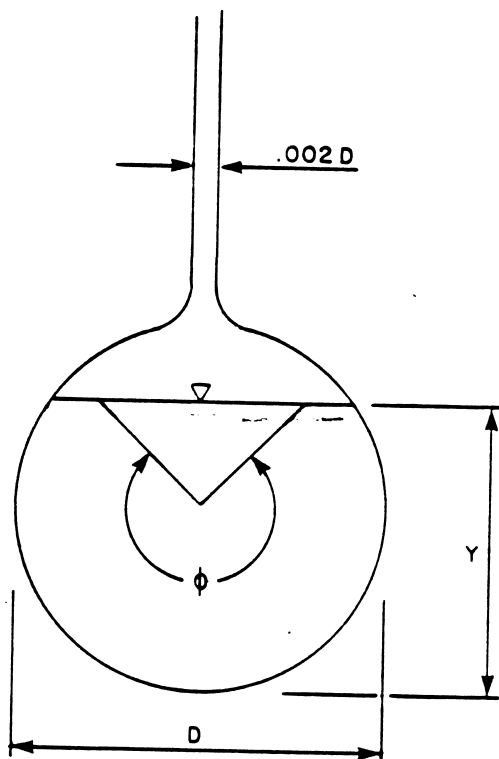


FIGURE 1 DISCRETE X - T SOLUTION DOMAIN



SECTIONAL VARIABLES

ϕ = CENTRAL ANGLE

Y = DEPTH OF FLOW

r = RADIUS

D = DIAMETER

W = WIDTH

A = AREA

P = PERIMETER

R = HYDRAULIC RADIUS

FIGURE 2 OPEN CHANNEL APPROXIMATION OF A CIRCULAR CONDUIT WITH A VIRTUAL CHIMNEY

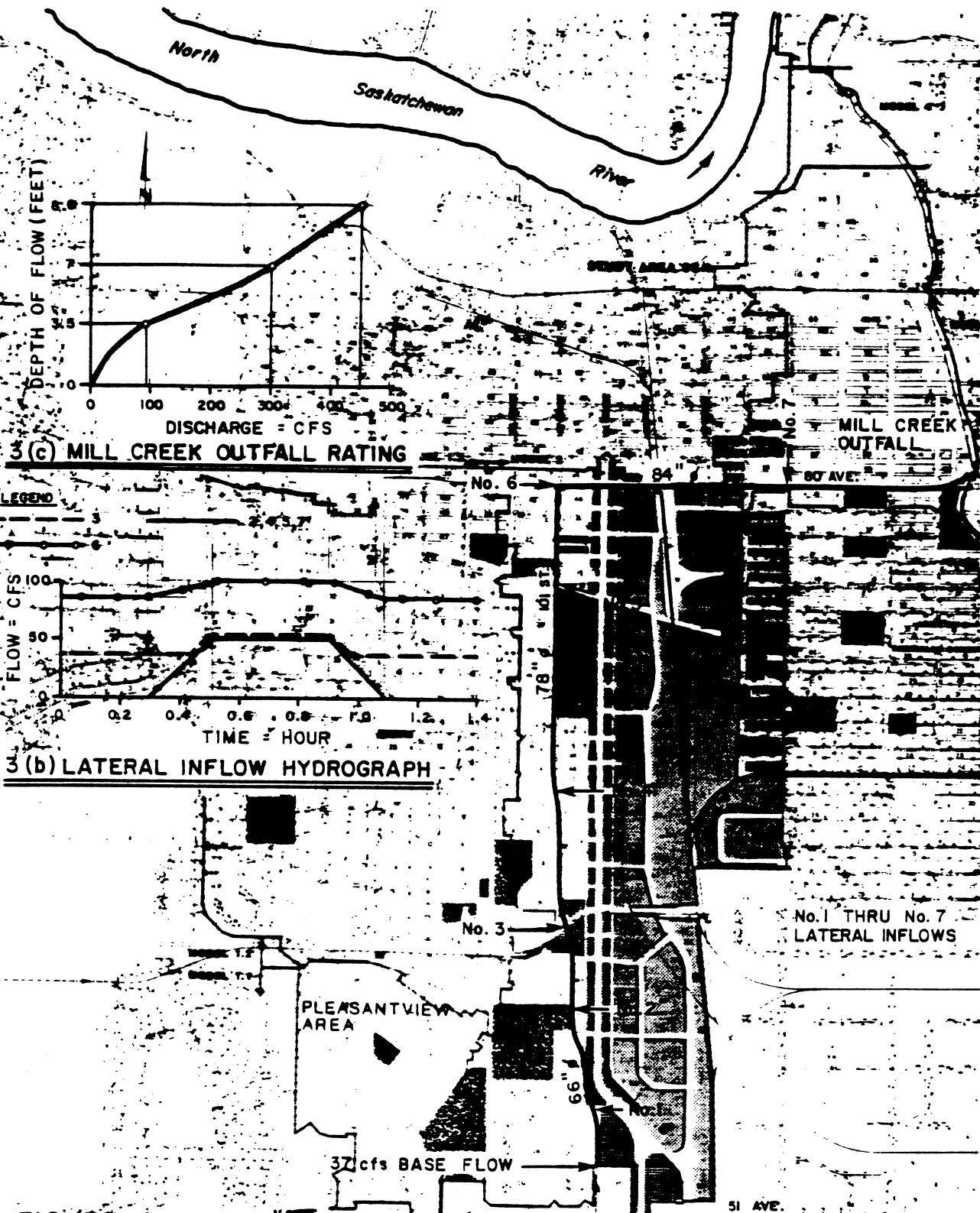


FIGURE 3 SOUTH SIDE COMBINED TUNNEL FROM 51 AVENUE TO MILL CREEK, CITY OF EDMONTON

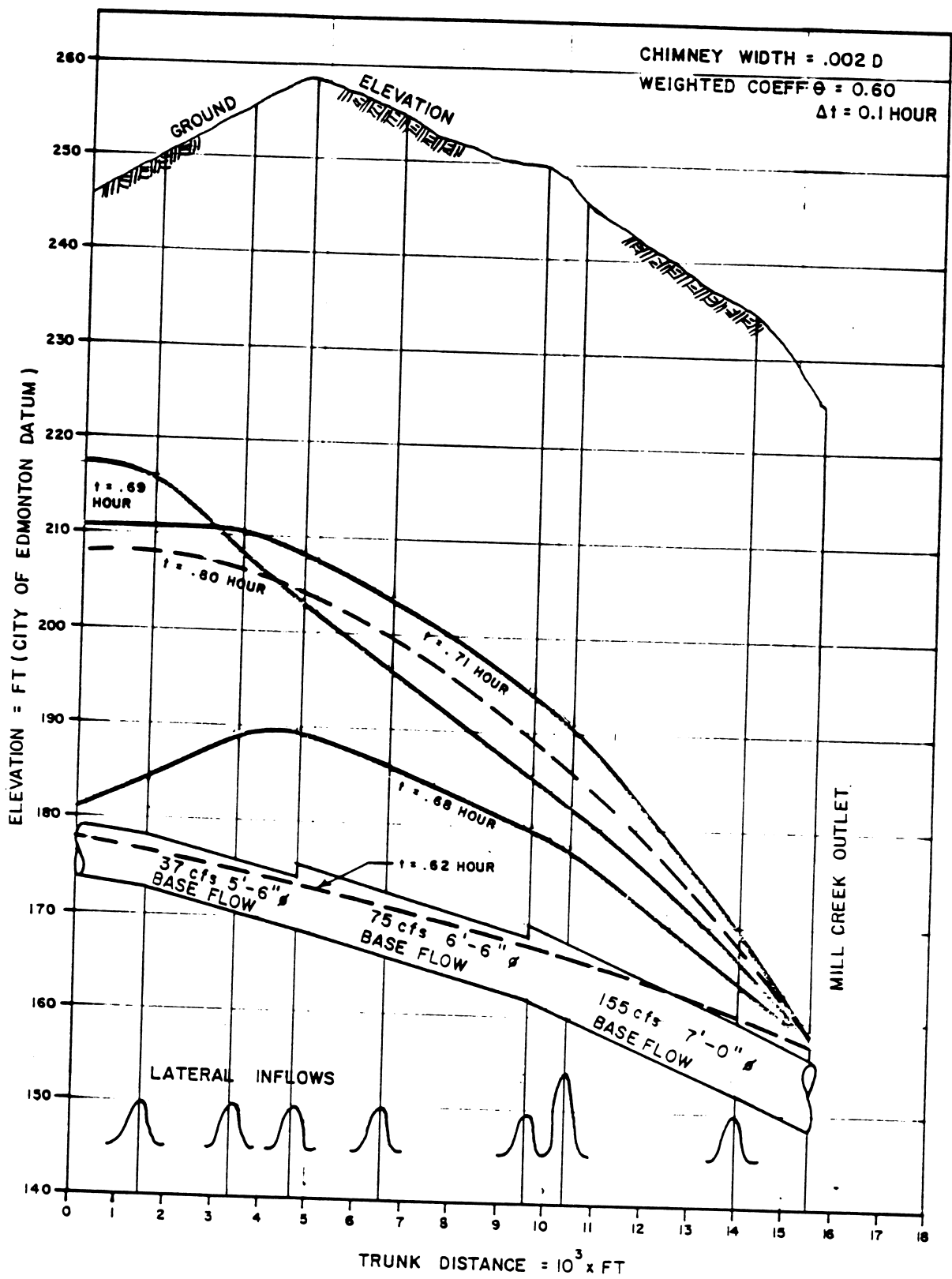


FIGURE 4 WATER LEVEL PROFILES COMPUTED BY SSP

